1.2 EOSC 579 - Chapter 4 - Lecture 1 : Sverdrup Circulation

1.2.1 Learning Goals

At the end of this lecture you will be able to:

- Sketch the observed ocean general circulation for a box model.
- Derive the Sverdrup balance and describe where in the ocean it is valid.

1.2.2 Ekman Pumping

Remember from EOSC 512 that due to variations in the Ekman Flux in the bottom boundary layer, there can be pumping into or out of the boundary layer. The vertical velocity at the top of the bottom boundary layer is:

$$w(top) = \frac{\delta \zeta_g}{2} \tag{1}$$

where $\delta = (2Av/f)^{1/2}$ and Av is the vertical eddy viscosity.

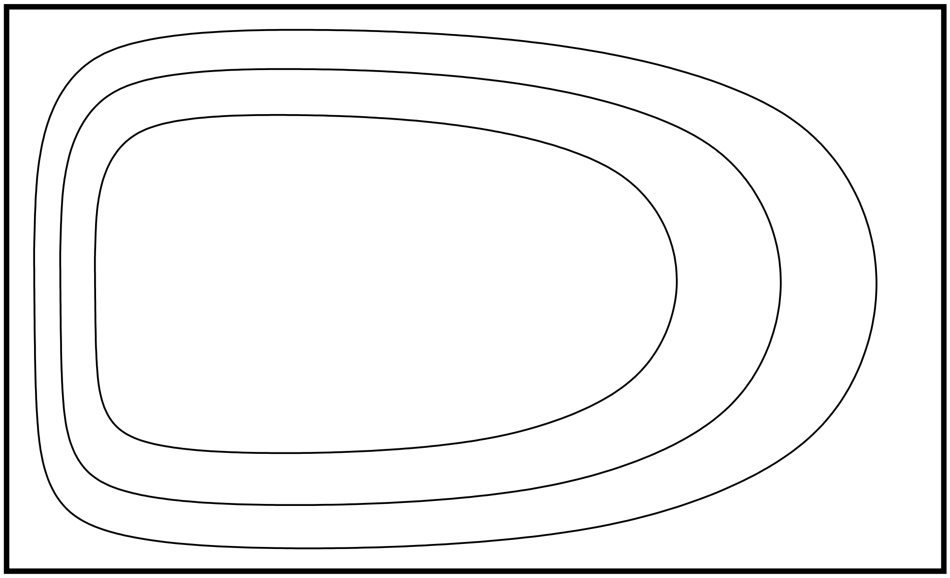
Also remember from EOSC 512 that due to variations in the Ekman flux in the surface boundary layer, there can be pumping into or out the boundary layer. The vertical velocity at the bottom of the top boundary layer is:

$$w(bot) = \frac{\nabla \times \vec{\tau}}{f \rho_o} \tag{2}$$

where $\vec{\tau}$ is the wind stress.

1.2.3 The Problem

Observations of the general circulation of the ocean show a series of gyres: the sub-polar gyres and the sub-tropical gyres. These gyres are un-symmetrical with stronger western boundary currents than eastern boundary currents. Its always the west regardless of the sign of the gyre (cyclonic for sub-polar gyres and anti-cyclonic for sub-tropical gyres) or the hemisphere.



Plan view of the streamlines of a model single gyre ocean.

Vorticity Equation

Return to the shallow water equations: Equation (1) in the first lecture of the course and consider the flow between the two Ekman layers. Here, vertical friction is not important and horizontal friction is rarely important so we will also neglect it. Again we will use the vorticity equation we derived (Equation (3) from the first lecture)

$$\frac{D_h}{Dt}(\zeta + f) + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)(\zeta + f) = 0 \tag{3}$$

Now consider the conservation of volume equation (**??**) and integrate from the "top" of the bottom Ekman layer to the "bottom" of the surface Ekman layer assuming a flat bottom:

$$\begin{aligned} \int_{top}^{bot} \frac{\partial w}{\partial z} dz &= -\int_{top}^{bot} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz \\ w(bot) - w(top) &= -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \\ \frac{\nabla \times \tau}{\rho_o f} - \frac{\zeta \delta}{2} &= -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \end{aligned} \tag{4}$$

where we have assumed that there is no vertical shear in the velocities except in the boundary layers. This assumption is consistent with the fact there is no shear in any of the forcing terms in momentum equations as the pressure gradients are independent of depth. We now substitute for horizontal convergence in (3) to get:

$$\frac{1}{(\zeta+f)}\frac{D_h}{Dt}(\zeta+f) = \frac{\nabla\times\tau}{\rho_o H f} - \frac{\zeta\delta}{2H} \tag{5}$$

Sverdrup Flow

Consider a steady-state solution for the middle of the gyre ($L = 1000$ km, $U = 0.1$ m s^{-1}, $f = 1\times10^{-4}$ s^{-1}, $\beta = 2\times10^{-11}$ (ms)$^{-1}$, $\delta = 40$ m, $H = 4000$ m). Scale. $\zeta = \mathcal{O}(U/L) = 10^{-7}$ s^{-1}. So in the first fraction in (5) we can ignore ζ in the denominator.

- $1(/f)\vec{u}\cdot\nabla\zeta = \mathcal{O}\left(U^2/fL^2\right) = 10^{-10}$ s^{-1}
- $\beta v/f = \mathcal{O}\left(\beta U/f\right) = 10^{-8}$ s^{-1}
- $\delta\zeta/2H = \mathcal{O}\left(\delta U/HL\right) = 10^{-9}$ s^{-1}

So in the mid-ocean get the Sverdrup balance

$$\beta v = \frac{1}{\rho H}\hat{k}\cdot\nabla\times\vec{\tau} \tag{6}$$

Or simply

$$v = \frac{1}{\rho\beta H}\hat{k}\cdot\nabla\times\vec{\tau} \tag{7}$$

Now geostrophic velocities are horizontally divergenceless so

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8}$$

and thus

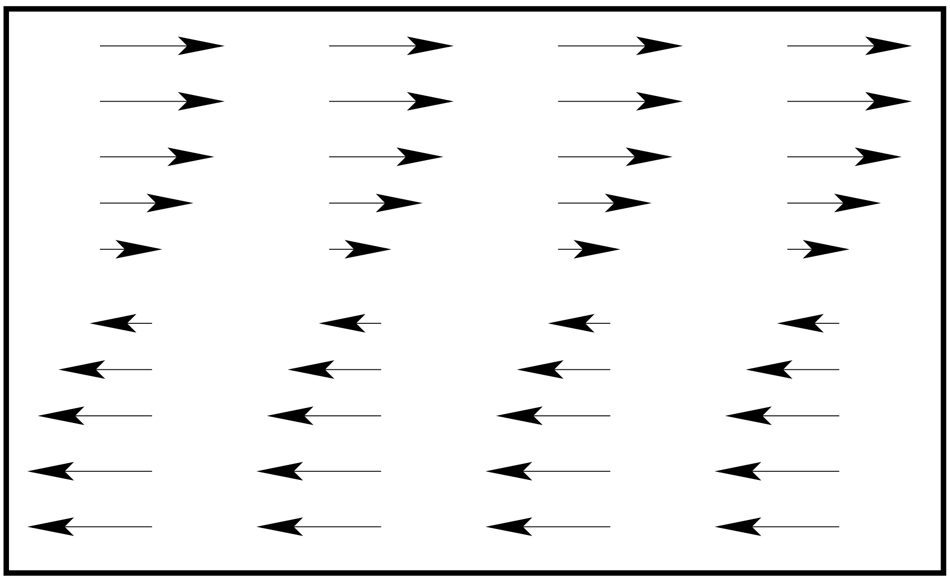
$$\begin{aligned} u(x) - u(x_o) &= -\int_{x_o}^{x}\frac{\partial v}{\partial y}dx \\ &= -\int_{x_o}^{x}\frac{\partial}{\partial y}(\hat{k}\cdot\nabla\times\vec{\tau})dx \end{aligned} \tag{9}$$

Assume the winds are purely east-west. Then $\tau_2 = 0$ and $\vec{\tau} = (\tau_1, 0)$. Then

$$v = \frac{-1}{\rho\beta H}\frac{\partial \tau_1}{\partial y} \qquad (10a)$$

$$u(x) - u(x_o) = \frac{1}{\rho\beta H}\int_{x_o}^{x}\frac{\partial^2 \tau_1}{\partial y^2}dx \qquad (10b)$$

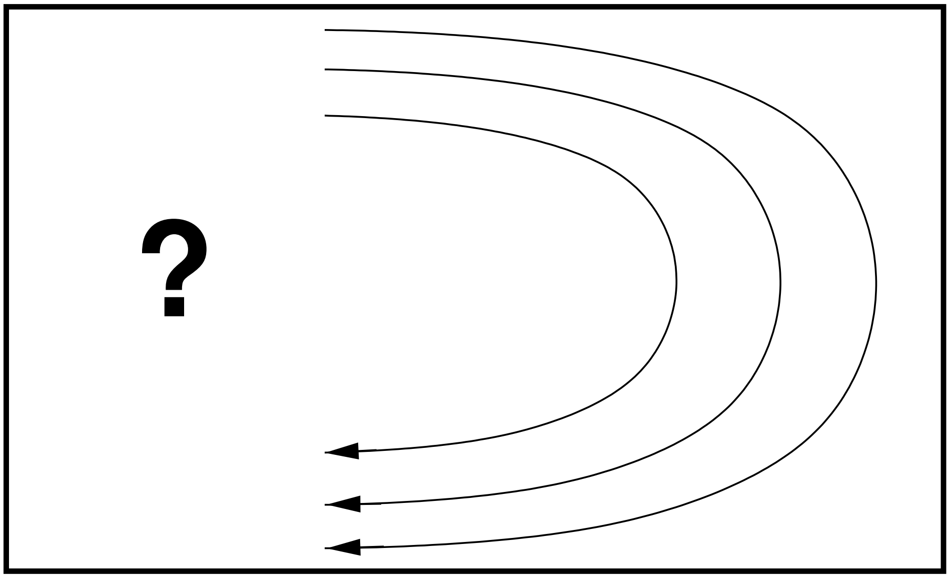
Choose $\tau_1 = -\tau_o \cos(\pi y/b)$ where b is the north-south length of the basin. Which looks like



Wind vectors over a northern hemisphere sub-tropical gyre

Note that $-\partial\tau_1/\partial y \propto -\sin(\pi y/b)$ so v is negative, largest at $y = b/2$ and zero at the both the north and south boundaries.

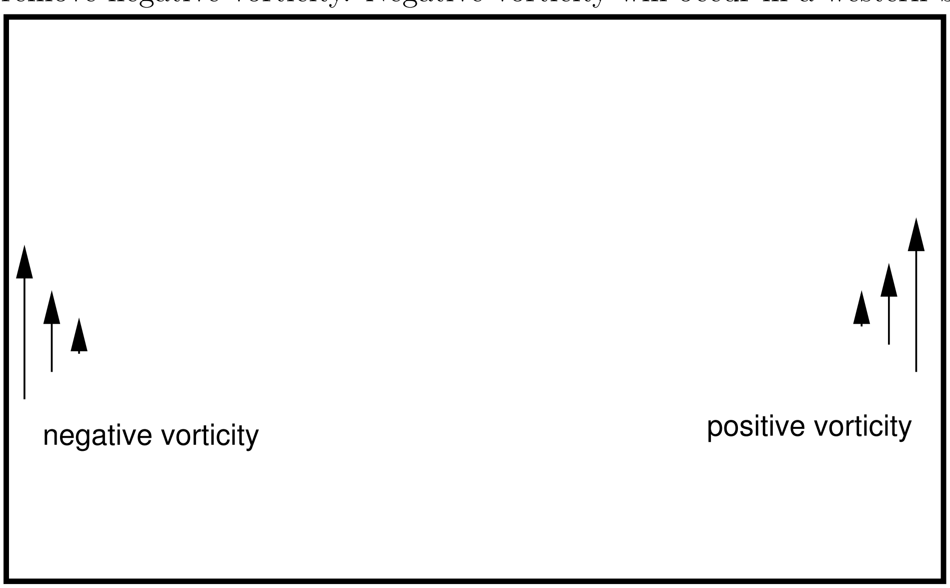
Note that $\partial^2\tau_1/\partial y^2 \propto \cos(\pi y/b)$. So one possible set of streamlines looks like



Sverdrup circulation for northern hemisphere sub-tropical gyre

where we have chosen to match the boundary conditions $u = 0$ on the eastern boundary. We cannot match boundary conditions on both eastern and western boundaries.... there must be a boundary layer. As v is uniformly southward, this boundary layer must move the fluid north.

In this boundary layer we expect either or both the nonlinear terms and the bottom friction to be important. Consider bottom friction. The boundary layer must move fluid north, so it must increase the planetary vorticity. For friction to increase vorticity, it must remove negative vorticity. Negative vorticity will occur in a western boundary current



Boundary Layer Vorticity for Northwards Flow