4.3 EOSC 579 - Chapter 4 - Lecture 3: Munk's General Circulation

4.3.1 Learning Goals

At the end of this lecture you will be able to:

- derive Munk's streamfunction equation
- sketch Munk's solution

4.3.2 What if its not bottom friction?

Wind puts energy into the flow. Something takes the energy out in order to have a steady solution. If the flow is insulated by stratification from the bottom, lateral (horizontal) friction effects must be important.

Consider a 500 m deep, homogeneous layer over a quiescent bottom layer. The upper layer is assumed to slip over the bottom layer. The linear, steady, momentum equations are:

$$-fv = -g\frac{\partial \eta}{\partial x} + A_H \nabla_h^2 u \tag{1a}$$

$$fu = -g\frac{\partial \eta}{\partial y} + A_H \nabla_h^2 v \tag{1b}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1c}$$

Form the vorticity equation by taking $\partial/\partial x$ of the second equation and subtracted $\partial/\partial y$ of the first equation:

$$f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \beta v = A_H \nabla^2 \zeta \tag{2}$$

Substitute the third equation

$$0 = f\frac{\partial w}{\partial z} - \beta v + A_H \nabla^2 \zeta \tag{3}$$

Integrate through the upper layer from its bottom, to the bottom of the upper Ekman layer.

$$\beta v = \frac{1}{\rho H}\hat{k} \cdot \nabla \times \vec{\tau} + A_H \nabla^2 \zeta \tag{4}$$

Substitute the streamfunction

$$\beta \frac{\partial \psi}{\partial x} = \frac{1}{\rho H}\hat{k} \cdot \nabla \times \vec{\tau} + A_H \nabla^4 \psi \tag{5}$$

This equation is Munk's equation.

4.3.3 Munk's Boundary Current

From our investigation of the Stommel Equation, we expect flow in the western boundary to be a balance between friction and the advection of planetary vorticity. Furthermore, we expect strong north-south velocities, weak east-west velocities and the scale across the boundary layer to be small compared the scale along the wall thus in the boundary layer we expect:

$$\beta \frac{\partial \psi}{\partial x} \approx A_H \frac{\partial^4 \psi}{\partial x^4} \tag{6}$$

With boundary condition that $u = -\partial\psi/\partial y = 0$, the wall is a streamline, but also $v = \partial\psi/\partial x = 0$, no-slip due to the viscosity.

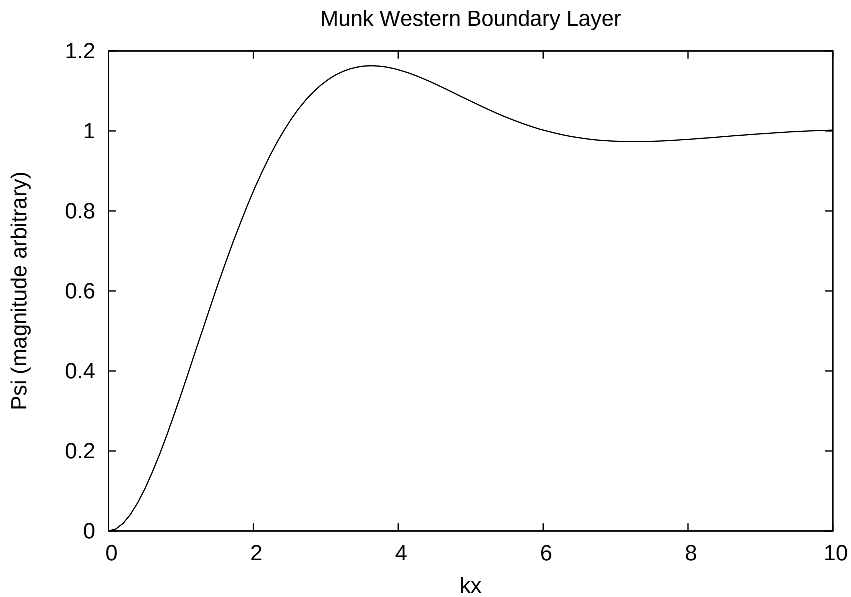
The approximate equation is a homogeneous ordinary differential equation with constant coefficients. Substitution of $\exp(\lambda x)$ gives $\lambda^3 = A_H/\beta$ so writing k as the real root, the three roots are k, $k/2(-1+\sqrt{3}i)$ and $k/2(-1-\sqrt{3}i)$.

So the solution is

$$\psi = A + B\exp(kx) + C\exp(-kx/2)\cos(\sqrt{3}kx/2) + D\exp(-kx/2)\sin(\sqrt{3}kx/2) \tag{7}$$

To solve the boundary conditions that the flow is a boundary layer, $B = 0$, to ensure that $\psi = 0$ at $x =$, $A = -C$ and to ensure that $\partial\psi/\partial x = 0$, $D = C/\sqrt{3}$. This gives a solution for the streamfunction out from the western boundary (Figure 4.1).

Figure 4.1 *Solution for the Munk Layer. SEA.*



Matching this solution to the Sverdrup interior solution (for $\tau_2 = 0$) one can find an approximate solution (see Pedlosky or Munk's original paper) for an arbitrary τ_1 (Plate 2).

Figure 2: *The streamlines of the circulation pattern for the Munk Solution, for a rectangular ocean driven by the stress distribution for the Pacific Ocean by Munk (1950). Provided under fair-dealing provision.*

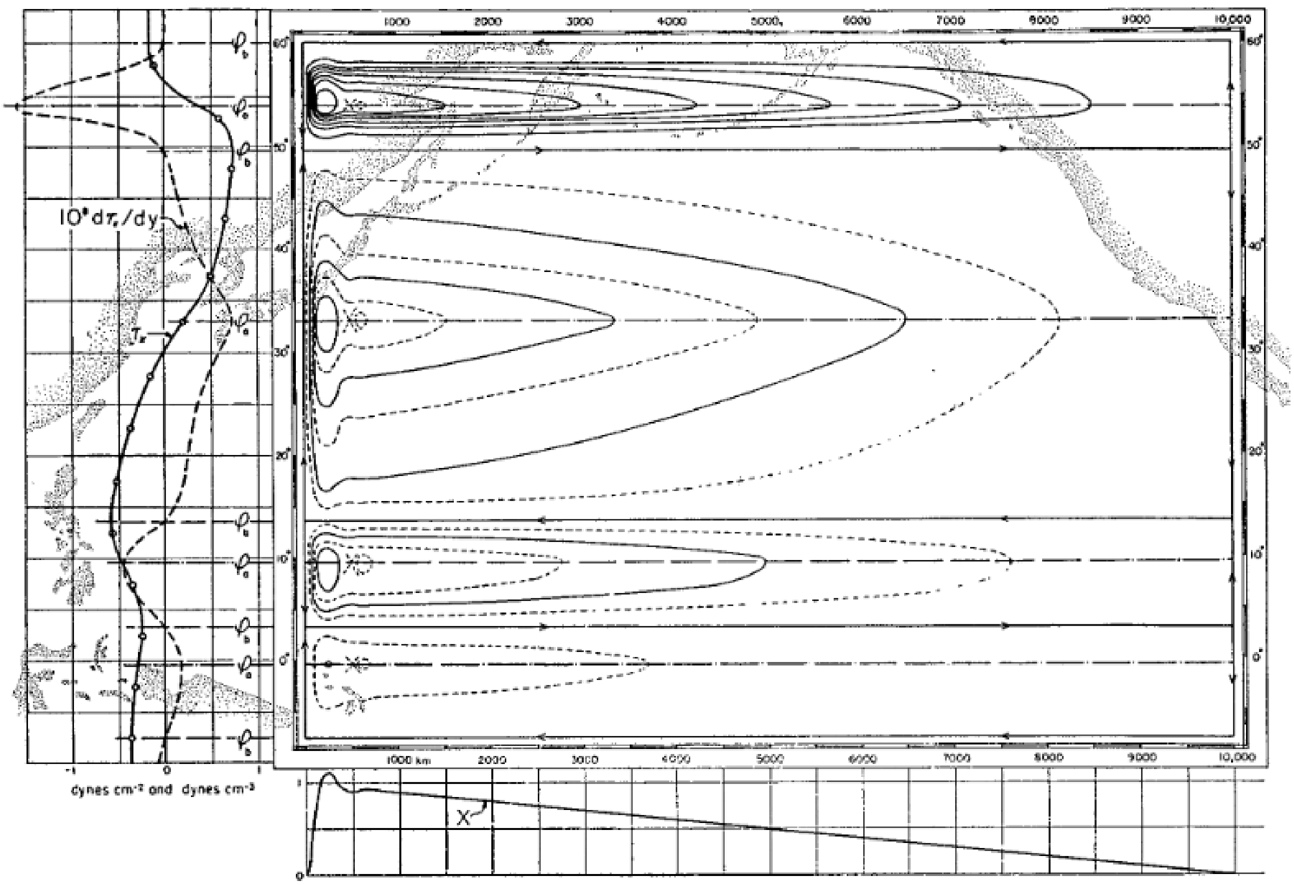


FIG. 2. The mean annual zonal wind stress $\tau_x(y)$ over the Pacific and its curl $d\tau_x/dy$ are plotted on the left, the function $X(x)$ on the lower part. These functions have been combined graphically according to equation (22) to give mass transport streamlines $\psi(x, y)$. The transport between adjacent solid lines is 10×10^{12} g sec^{-1}, or 10 million metric tons sec^{-1}. The total transport between the coast and any point x, y is $\psi(x, y)$. The chart of the Pacific has a uniform distance scale throughout. In the relatively narrow northern portion, the transport is greatly exaggerated.