EOSC 579 - Chapter 4 - Lecture 4: Fofonoff and Moore's Solutions

4.3.1 Learning Goals

At the end of this lecture you will be able to:

- derive Fofonoff's streamfunction equation
- sketch Fofonoff's solution
- discuss Moore's solution

4.3.2 Fofonoff's Solution

What about the nonlinear advection terms? They are not small in the Western boundary, particular where it starts and ends.

Fofonoff consider the 'natural modes' of flow in a homogeneous, flat-bottom ocean on a β-plane. So assume no friction, no forcing. In this case the total vorticity is conserved so

$$\frac{D}{Dt}(\zeta_a) = 0 \tag{1}$$

where $\zeta_a = f + \zeta$. Assuming steady flow:

$$\vec{u}_h \cdot \nabla \zeta_a = 0 \tag{2}$$

So along the flow the vorticity is conserved. Or the vorticity is constant along a streamline.

Thus we can write $\zeta_a = F(\psi)$ where F is some unknown function. Fofonoff assumed ζ_a is a linear function of ψ. Thus

$$\zeta_a = f + \zeta = f + \nabla^2 \psi = c_o + c_1 \psi \tag{3}$$

or

$$\nabla^2 \psi - c_1 \psi = -f_o - \beta y + c_o \tag{4}$$

Note that the asymmetry is in y not x. Fofonoff found solutions of this equation in a rectangular basins. His modes look like different depending on the wind direction: cyclonic

(Figure **??**) or anti-cyclonic (Figure **??**).

Figure 4.1 *Solution for Cyclonic Fofonoff Flow. SEA.*

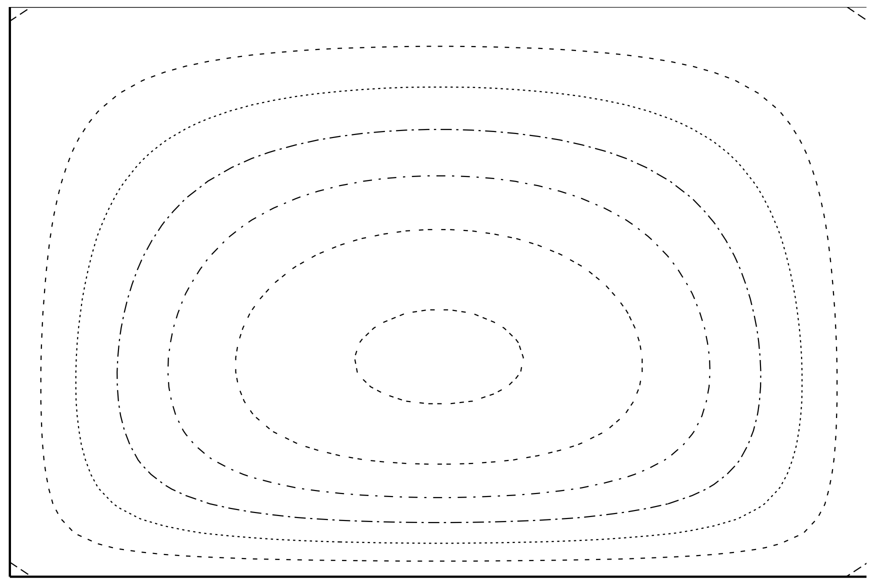
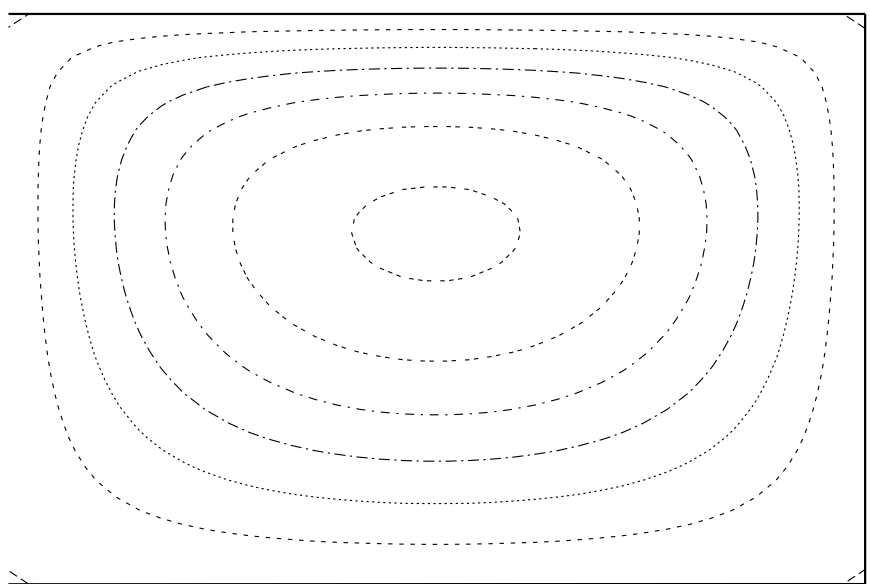


Figure 4.2 *Solution for Anti-cyclonic Fofonoff Flow. SEA.*



Note in both cases we get strong eastward flow, as is observed as the Gulf-Stream leaves the coast of the United States.

4.3.3 Moore's Solution

Moore used both the inertial terms and lateral viscosity (combination of Munk and Fofonoff) but found a numerical solution only for "weak" flows. For a Reynolds number of 5 (Plate **??**), he gets a strong western boundary current, weak currents on the eastern side and southern side, and a strong, oscillatory current on the western part of the northern side. His Reynolds number is defined as:

$$Re = \frac{U^{3/2}}{A_H \beta^{1/2}} \tag{5}$$

Figure 4.3 *Streamlines of the circulation pattern than Moore found for a homogeneous fluid with a constant cosine wind-stress under weakly nonlinear conditions. Figure 1 from D.W. Moore, 1963, Deep-Sea Research. Vol 10 pg 735-747. Copyright: Elsevier. Provided under fair dealing.*

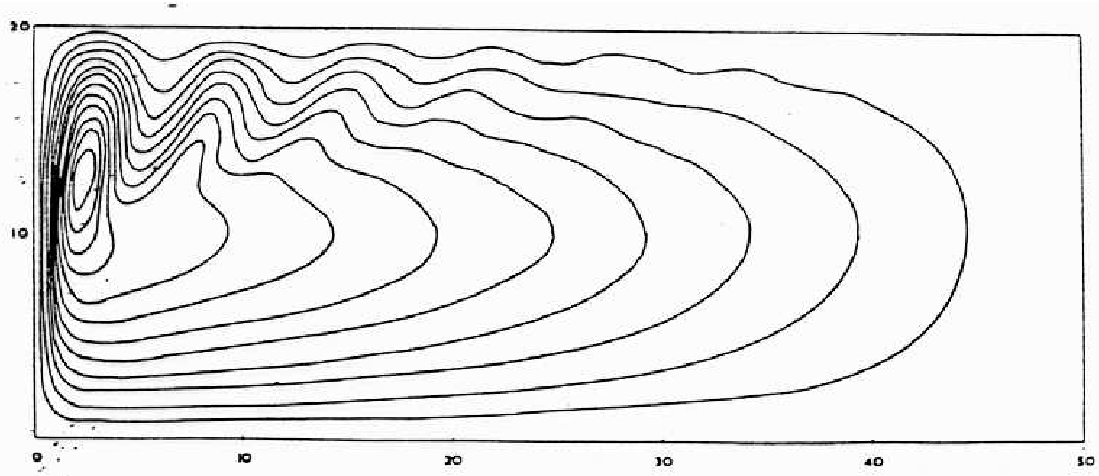


Figure 5.5 Contours of the streamfunction in a homogeneous ocean driven by a wind stress of the form $-\cos \pi y/M$ as derived by Moore [1963]. An Oseen approximation for the nonlinear terms with a mean current $U(y) \propto \cos \pi y/M$ was used. The wavy contours in the north half-basin are standing Rossby waves imbedded in the mean velocity field.