HW 7 Answer Key

ATSC 201 Fall 2024
Chapter 4: A1f, A4f, A13f, A17f, A25f, E17

Total marks out of 40

Chapter 4

A1f) (5 marks)

Compare the saturation vapor pressures (with respect to liquid water) calculated with the Clausius-Clapeyron equation and with Tetens' formula, for T(°C): e) 20

Given: T = 20 °C 293.15 K

Use eqn 4.1a (Clausius Clapeyron eqn)

$$e_s = e_o \cdot \exp\left[\frac{L}{\Re_v} \cdot \left(\frac{1}{T_o} - \frac{1}{T}\right)\right]$$

where Rv = 461 J/(K*kg)
To = 273.15 K
eo = 0.611 kPa
L = Lv (for liquid water) = 2.50E+06 J/kg

Use eqn 4.2 (Teten' formula)

$$e_s = e_o \cdot \exp\left[\frac{b \cdot (T - T_1)}{T - T_2}\right]$$

where b = 17.2694
eo = 0.611 kPa
T1 = 273.15 K
T2 = 35.86 K

Clausius-Clapeyron:

es =	2.368 kPa

Teten's:

es =	2.339 kPa

Clausius-Clapeyron's saturation vapor pressure is slightly larger than Teten's.

Check: Units ok. Physics ok.
Discussion: The Clausius-Clapeyron gives a saturation vapor pressure slightly

larger to Teten's formula for a temperature of 25°C. This is consistent with the graph on p.89 from the sample application. Teten's formula accounts for L varying slightly, but it appears the difference is usually negligible.

A4f)
(18 marks)

Calculate the values of es (kPa), r (g/kg), q (g/kg), ρv (g/m^3), RH (%), Td (°C), LCL (km), Tw (°C), rs(g/kg), qs (g/kg), and ρvs (g/m^3), given the following atmospheric state: ef) P = 90kPa, T = 5°C, e = 0.2kPa.

Given:	P =	90	kPa	
	T =	5	degC	278.15 K
	e =	0.2	kPa	

Find:	es =	?	kPa
	r =	?	g/kg
	q =	?	g/kg
	ρv =	?	g/m^3
	RH =	?	%
	Td =	?	degC
	LCL =	?	km
	Tw =	?	degC
	rs =	?	g/kg
	qs =	?	g/kg
	ρvs =	?	g/m^3

Use eqn. 4.1a:

$$e_s = e_0 \cdot \exp\left[\frac{L}{\Re_v} \cdot \left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

where eo = 0.611 kPa
To = 273.15 K
Lv/Rv = 5423 K

es = 0.87 kPa

Use eqn 4.4: r = ε*e/ (P - e)
where ε = 622 g/kg 0.622 g/g

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r =	**1.39 g/kg**

Use eqn. 4.7: q = ε*e/P

q =	**1.38 g/kg**

Use eqn. 4.10: ρv = e/(Rv * T)
where Rv = 4.61E-04 kPa*m^3/(K*g)

ρv =	**1.56 g/m^3**

Use eqn. 4.14a: RH =(e/ es) * 100

RH =	**22.91 %**

Use eqn. 4.15a:

$$T_d = \left[\frac{1}{T_o} - \frac{\Re_v}{L}\cdot\ln\left(\frac{e}{e_o}\right)\right]^{-1}$$

where eo = 0.611 kPa
To = 273 K
Rv/Lv = 1.84E-04 /K

Td = 258.47 K

Td =	**-14.68 degC**

Use eqn. 4.16a: LCL = a*(T-Td)
where a = 0.125 km/degC

LCL =	**2.46 km**

To find wet bulb temperature using Normand's Rule:
Use eqn 4.21: Tw = T_LcL + Γs*LCL

Find Γs with eqn 4.37a:

$$\Gamma_s = \frac{|g|}{C_p}\cdot\frac{\left(1+\frac{r_s\cdot L_v}{\Re_d\cdot T}\right)}{\left(1+\frac{L_v^2\cdot r_s\cdot\varepsilon}{C_p\cdot\Re_d\cdot T^2}\right)}$$

Use eqn. 3.3: cp = cpd*(1+1.84*r)
where cpd = 1004 J/(kg*K)

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r (g/g) =	0.00139	
cp =	1006.55915	J/(kg*K)

g/cp =	9.74	K/km	(I calculated some intermediate
Lv/cp =	2483.71	K	values to make the huge eqn nicer
Lv/Rd =	8707.77	K	to input)

Γs = 5.63 K/km

Find T_LcL with eq 4.20: T_LcL = T - Γd*LCL

where Γd = 9.8 K/km

T_LcL = 254.04 K

Tw = 267.89 K

Tw = -5.26 degC

Use eqn 4.5: rs = (ε*es)/(P-es)

rs = 6.09 g/kg

rs = 0.00609279 g/g

Use eqn. 4.8: qs = (ε*es)/P

qs = 6.03 g/kg

Use eqn. 4.12: ρvs = es/(Rv*T)

ρvs = 6.81 g/m^3

Check: Units ok. Physics ok.

Discussion:

Because the air at this level is unsaturated, any liquid droplets in the air will tend to evaporate. Imagine leaving a wet towel in a room; it will eventually dry up due to evaporation. This evaporation causes cooling in the wet towel. The wet-bulb temperature represents the temperature of the wet towel after the evaporative cooling. Therefore, you would always expect the wet-bulb temperature to be cooler than the dry-bulb temperature in unsaturated air.

A13f)
(3 marks)

For air at sea level, find the total water mixing ratio for a situation where: f) r= 4 g/kg, rL = 1 g/kg.

Given: r = 4 g/kg
rL = 1 g/kg

Find: rT = ? g/kg

Use eqn 4.32a:
rT = rs + rL + ri

Need ri:
Assume ri = 0 g/kg

rT = 5 g/kg

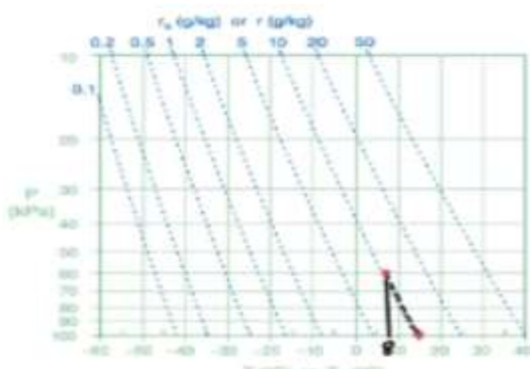
Check: Units ok. Physics ok.
Discussion: Since we assume there is no ice presents in our parcel, the amount of water vapor in the air is simply the sum of the water vapor and the liquid present in our parcel.

A17f)
(3 marks)

Given an air parcel starting at 100kPa with dewpoint (degC) given below, use Fig. 4.7 to find the parcel's final dewpoint (degC) if it rises to a height where P = 60kPa. f) 15

Given:
Pi = 100 kPa
Tdi = 15 degC
Pf = 60 kPa

Find:
Tdf = ? degC



See attached Fig. 4.7.

Tdf =	**8 degC**

Check: Units ok. Physics ok.
Discussion: The dew point temperature decreases with height because air expands and cools adiabatically - just like with regular temperature.

A25f)
(3 marks)

Given an air parcel that starts at a height where P = 100kPa with T = 25degC and r = 12 g/kg (ie. it is initially unsaturated). After rising to its final height it has an rL (g/kg) value listed below. Assuming no precipitation falls out, find the value for r (g/kg) for this now-saturated air parcel. f) 3

Given:	P_initial =	100	kPa
	T_initial =	25	degC
	r_initial =	12	g/kg
	rL_final =	3	g/kg
	rL_initial =	0	g/kg

Find: r_final = ? g/kg

Use eqn 4.35b:

$$(r + r_L)_{initial} = (r + r_L)_{final} \qquad \bullet(4.35b)$$

Re-arrange: r_final = r_initial + rL_initial - rL_final

r_final =	**9 g/kg**

Check: Units ok. Physics ok.
Discussion: In this situation, we are assuming that the water vapor condenses

into liquid form, and the liquid droplets are suspended in the air. This way, we can assume that the total mixing ratio remains constant, as water is not lost through precipitation.

E17)
(8 marks)

Create a thermo diagram using a spreadsheet to calculate isohumes (for r = 1, 3, 7, 10, 30 g/kg) and dry adiabats (for θ = -30, -10, 10, 30 °C)**, all plotted on the same graph vs P on an inverted log-scale similar to Figs. 3.3 and 4.7.**

Find: Plot isohumes on a thermo diagram.

Use: $$T_d = \left[\frac{1}{T_o} - \frac{\Re_v}{L}\ln\left(\frac{r \cdot P}{e_o \cdot (r+\varepsilon)}\right)\right]^{-1}$$ eqn 4.15b

Or: $$T = \left[\frac{1}{T_o} - \frac{\Re_v}{L_v}\cdot\ln\left\{\frac{r_s \cdot P}{e_o \cdot (r_s+\varepsilon)}\right\}\right]^{-1}$$ eqn 4.36

eo = 0.611 kPa
Rv/Lv = 1.84E-04 1/K
To = 273 K
ε = 0.622 g/g

P (kPa)	T for r=1	T for r=3	T for r=7	T for r=10	T for r=30
10	-42.27	-30.99	-21.56	-17.39	-3.84
20	-35.26	-23.27	-13.21	-8.75	5.75
30	-30.96	-18.52	-8.06	-3.43	11.68
40	-27.81	-15.03	-4.28	0.49	16.05
50	-25.31	-12.27	-1.28	3.60	19.52
60	-23.23	-9.96	1.23	6.19	22.43
70	-21.44	-7.98	3.38	8.43	24.93
80	-19.88	-6.24	5.27	10.39	27.13
90	-18.48	-4.68	6.97	12.15	29.10
100	-17.21	-3.28	8.50	13.73	30.89

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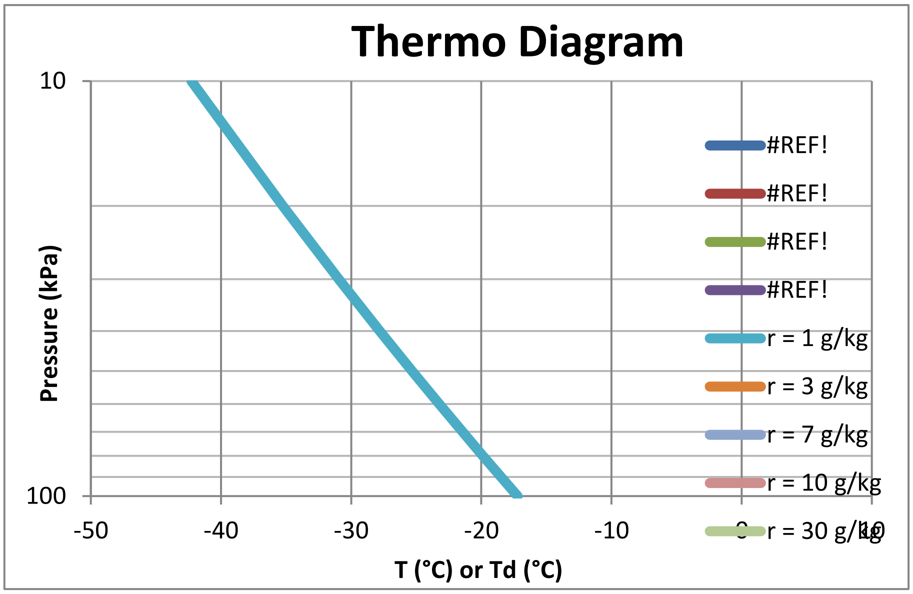
r (g/g)
0.001
0.003
0.007
0.01
0.03



Check: Units ok. Physics ok. Looks like fig 4.7.

Discussion: When we are given the air temperature and the dew point temperature of a parcel, we can use the thermodynamic diagram to find its lifting condensation level by following the dry adiabat from the temperature point, and the isohume from the dew-point temperature point.