

HW 6 Answer Key

**ATSC 201 Fall 2024**

**Chapter 3: A10e, A14e, A15, A20e, A27e, E1**

**Total mark out of 28**

HW 6 Answer Key

Chapter 3

A10f)  
(3.5 marks)

Given air with temperature and altitude as listed below, use formulas (not thermo diagrams) to calculate the potential temperature. Show all steps in your calculations. f)  $z(\text{m}) = 6000$ ,  $T(\text{degC}) = -50$

Given:  $z = 6000 \text{ m}$   
 $T = -50 \text{ deg C}$

Find:  $\theta(z) \quad ? \quad \text{deg C}$

Use eqn 3.11:

$$\theta(z) = T(z) + \Gamma_d \cdot z \quad \bullet(3.11)$$

Use eqn. 3.8:

$$\Gamma_d = 9.8 \text{ K km}^{-1} = 9.8 \text{ }^\circ\text{C km}^{-1}$$

Convert  $z(\text{m})$  into  $z(\text{km})$ :

$z = 6 \text{ km}$

$\theta(z) = 8.80 \text{ deg C}$   
 $281.95 \text{ K}$

Check: Units ok. Physics ok.

Discussion: This is the temperature that a parcel at height of 6km would have if it were brought down to  $z=0$  dry adiabatically. Potential temperature is constant through dry adiabatic processes.

$$\theta_v = \theta \cdot [1 + (a \cdot r)] \quad (3.13)$$

A14f)  
(3 marks)

Instead of equations, use the Fig 3.4 to find the actual air temperature (degC) given: f)  $P(\text{kPa}) = 50$ ,  $\theta(\text{degC}) = 10$ .

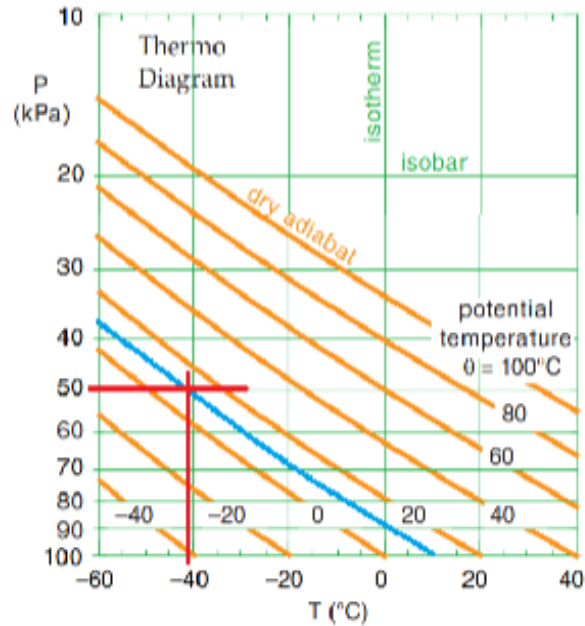
Given:  $P = 50 \text{ kPa}$

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$\theta = 10 \text{ degC}$

Find:  $T = ? \text{ deg C}$

Using thermo diagram:



<b>T =</b>	<b>-40 degC</b>
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Check: Units ok.

Discussion: Thermo diagrams are easy to use as seen in this question to convert between potential temperature and air temperature

**A15)**  
(7.5 marks)

**Use a spreadsheet to calculate and plot a thermo diagram similar to Fig. 3.4 but with isotherm grid lines every 10degC, and dry adiabats for every 10degC from -50degC to 80degC.**

Use eqn. 3.10:

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\mathcal{R}_d / C_p} \quad \bullet(3.10)$$

$R_d/c_p = 0.28571$

Each T(degC) column is a dry adiabat

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P (kPa)	T (degC)	T (degC)	T (degC)	T (degC)	T (degC)	T (degC)
10	-157.49657	-152.31705	-147.13752	-141.958	-136.77847	-131.59895
20	-132.20017	-125.88627	-119.57238	-113.25848	-106.94459	-100.6307
30	-114.9068	-107.81742	-100.72804	-93.638662	-86.549281	-79.4599
40	-101.36359	-93.66689	-85.97019	-78.27349	-70.57679	-62.880089
50	-90.064672	-81.861294	-73.657916	-65.454538	-57.25116	-49.047782
60	-80.282809	-71.640783	-62.998756	-54.35673	-45.714703	-37.072676
70	-71.605399	-62.574251	-53.543103	-44.511955	-35.480807	-26.449659
80	-63.773494	-54.391139	-45.008785	-35.62643	-26.244076	-16.861721
90	-56.612838	-46.909378	-37.205918	-27.502458	-17.798998	-8.0955374
100	-50	-40	-30	-20	-10	0.0000001

T (degC)	T (degC)	T (degC)	T (degC)	T (degC)	T (degC)	T (degC)
-126.41942	-121.23989	-116.06037	-110.88084	-105.70132	-100.52179	-95.342265
-94.316802	-88.002909	-81.689015	-75.375121	-69.061227	-62.747333	-56.433439
-72.370519	-65.281138	-58.191757	-51.102376	-44.012995	-36.923614	-29.834234
-55.183389	-47.486689	-39.789989	-32.093289	-24.396589	-16.699889	-9.0031894
-40.844405	-32.641027	-24.437649	-16.234271	-8.0308929	0.17248504	8.37586297
-28.43065	-19.788624	-11.146597	-2.5045705	6.13745598	14.7794825	23.421509
-17.418511	-8.3873626	0.64378539	9.67493343	18.7060815	27.7372295	36.7683775
-7.4793664	1.90298812	11.2853427	20.6676972	30.0500518	39.4324063	48.8147608
1.6079227	11.3113829	21.014843	30.7183032	40.4217634	50.1252235	59.8286837
10	20	30	40	50	60	70

T (degC)

-90.16274

-50.119545

-22.744853

-1.3064893

16.5792409

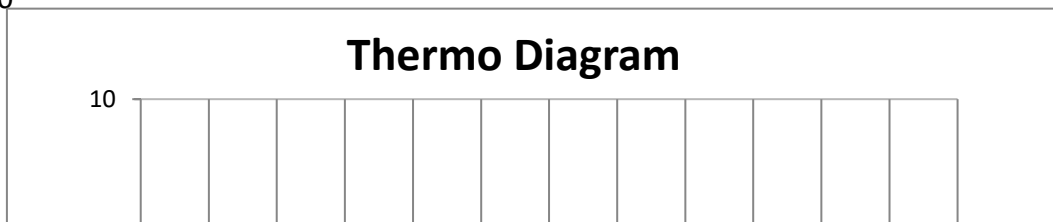
32.0635355

45.7995256

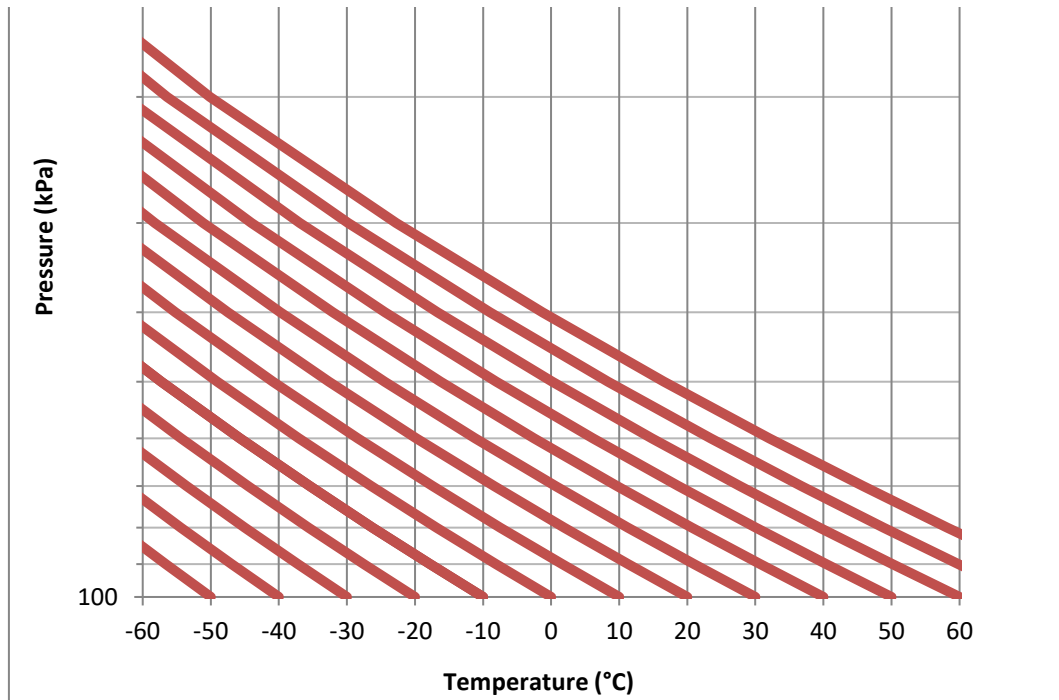
58.1971154

69.5321439

80



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Check: Units ok. Physics ok.

Discussion:

A thermo diagram is useful in many ways, including determining quickly the stability of different layers in the atmosphere, based on temperature soundings, to predict whether clouds and thunderstorms will form.

**A20f)**  
(3 marks)

**Find the effective surface turbulent heat flux ( $^{\circ}\text{C}\cdot\text{m/s}$ ) over a forest for wind speed of 10 m/s, air temperature of  $20^{\circ}\text{C}$ , and surface temperature ( $^{\circ}\text{C}$ ) of: f) 26.**

Given:       $T_a =$                        $20^{\circ}\text{C}$   
                   $T_s =$                        $26^{\circ}\text{C}$   
                   $M =$                          $10\text{ m/s}$

Find:         $FH =$             ?                       $^{\circ}\text{C}\cdot\text{m/s}$

Use eqn. 3.35:

$$FH = CH \cdot M \cdot (T_s - T_a)$$

where  $CH =$              $2.00\text{E-}02$  for forests

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<b>FH =</b>	<b>1.20 °C*m/s</b>
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Check: Units ok. Physics ok.

Discussion: For every degree Celsius difference between the surface temperature and the air temperature, the effective surface turbulent heat flux increases by 0.2 degC\*m/s for a given wind speed of 10m/s. The stronger the winds are, the greater the increase in the effective surface turbulent heat flux.

**A27f)**  
(4 marks)

<p><b>Given a pre-storm environment where the temperature varies linearly from 25°C at the Earth's surface to -60°C at 11km (tropopause). What is the value of the vertical gradient of turbulent flux (K/s) for an altitude (km) of: f) 2.5km</b></p>
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Given:	$z_T =$	11 km	
	$z =$	2.5 km	
	$\Gamma_{sa} =$	6.5 K/km	
	$\Delta t =$	1 hr	3600 s

Find:  $\Delta F_z / \Delta z =$  ? K/s

First find initial lapse rate using eqn. 3.6:

$$\Gamma = \Gamma = -\frac{T_2 - T_1}{z_2 - z_1} = -\frac{\Delta T}{\Delta z} \quad (3.6)$$

where $T_2 =$	-60 degC	213.15 K
$T_1 =$	25 degC	298.15 K

$z_2 = z_T =$	11 km
$z_1 =$	0 km

$\Gamma_{ps} = 7.72727273 \text{ degC/km} = \text{K/km}$

Now use eqn. 3.43:

$$\frac{\Delta F_z \text{ turb}}{\Delta z} \approx \frac{z_T}{\Delta t} \cdot [\Gamma_{ps} - \Gamma_{sa}] \cdot \left( \frac{1}{2} - \frac{z}{z_T} \right) \quad (3.43)$$

<b><math>\Delta F / \Delta z =</math></b>	<b>0.00102 K/s</b>
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Check: Units ok. Physics ok.

Discussion: The vertical turbulent flux gradient mixes the pre-storm air until the temperature profile returns to that of the standard atmosphere

E1)  
(7 marks)

**Assume that 1kg of liquid water initially at 15°C is in an insulated container. Then you add 1kg of ice into the container. The ice melts and the liquid water becomes colder. Eventually a final equilibrium is reached. Describe what you end up with at this final equilibrium?**

Given:  $T_i = 15\text{ }^\circ\text{C}$   
 $m_{\text{water}} = 1\text{ kg}$   
 $m_{\text{ice}} = 1\text{ kg}$   
 $L_f = 334\text{ kJ/kg} \quad 3.34\text{E}+05\text{ J/kg}$   
 $C_{\text{liquid}} = 4.218\text{ kJ/(kg*degC)} \quad 4218\text{ J/(kg*degC)}$   
Assume ice has an initial temperature of 0°C

An insulated container is an isolated system. This means that the water and ice only give/take energy from each other, not their surroundings. So equilibrium is reached when the water reaches a temperature of 0°C and the ice stops melting, OR, when all ice has melted and the water is still above 0°C.

Find energy required to melt all of the ice:

$$\Delta Q_e = \Delta m_{\text{ice}} * L_f$$

$$\Delta Q_e = 334000\text{ J}$$

Use eqn. 3.4b to find how much energy released when water cools to 0°C.

This is how much energy we have available to melt the ice.

$$\Delta Q_h = m_{\text{water}} * C_{\text{liquid}} * \Delta T$$

$$\text{where } \Delta T = -15\text{ }^\circ\text{C}$$

$$\Delta Q_h = -63270\text{ J}$$

The water will cool to 0°C and release 63270 J of energy (sensible heat). This energy is transferred to the ice, but it is not enough energy to melt ALL the ice (which would take 334000 J). But SOME of the ice will melt.

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Now find  $\Delta m_{\text{ice}}$  (how much of the ice will melt) given  $\Delta q_e = 63270 \text{ J}$

$$\begin{aligned}\Delta m_{\text{ice}} &= \Delta q_e / L_f && \text{(just equation 3.1 but rearranged)} \\ \Delta m_{\text{ice}} &= -0.189 \text{ kg}\end{aligned}$$

which means 0.189kg of ice has melted before the water and ice are in an equilibrium state at  $0^\circ\text{C}$ .

Finally find how much ice is left as solid:

$$\begin{aligned}\Delta m_{\text{ice\_unmelted}} &= m_{\text{ice\_initial}} - m_{\text{ice\_melted}} \\ &= 1\text{kg} - 0.189\text{kg} \\ &= 0.811 \text{ kg}\end{aligned}$$

And how much water we have at the end:

$$M_{\text{water}} = 1 \text{ kg} + 0.189 \text{ kg} = 1.189 \text{ kg}$$

final state:

**$T = 0^\circ\text{C}$  for the water and ice.**  
 **$m_{\text{ice}} = 0.811\text{kg}$  of ice is still in the water.**  
 **$M_{\text{water}} = 1.189 \text{ kg}$  of water is in the container.**

Check: Units ok. Physics ok.

Discussion: In a non-insulated container, the ice would fully melt and equilibrium would not be reached until the water is the same temperature as the surrounding air.